A New Design Method of Wavelet Filter

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Abstract: FFT and wavelet transform are the two major methods for harmonic detection. Wavelet transform is a very effective time-frequency analysis tool for analyzing non steady state signals. It overcomes the shortcomings of the Fourier Transform which is completely localized in the frequency domain and completely non-local in the time domain. In this paper, we describe the application of wavelet transform for harmonic detection, and then propose a new wavelet filter design method. The filter bank constructed by this method is consistent with the arrangement order of the nodes in the decomposition of wavelet packet, so by this method, we can easily determine the corresponding frequency band of the signal.

1. Introduction

FFT and wavelet transform are the two main methods currently used for harmonic detection. FFT is relatively mature, for steady-state harmonics, FFT is undoubtedly the best harmonic detection method [1]. It can accurately and directly obtain the information of amplitude and phase for each steady-state harmonic. The number of algorithm calculations is small, ensuring the real-time measurement. However, in practical power systems, due to the inherent non-linearity, randomness, distribution, and non-stationary complexity of harmonics, FFT algorithm is difficult to accurately measure harmonics [2].

Wavelet Transformation (WT) is a very effective time-frequency analysis tool which developed for the limitations of Fourier transform in analyzing non-steady-state signals [3]. It overcomes the disadvantages of Fourier transform which is full localization in the frequency domain and non localization in the time domain. It has locality in both the frequency and time domains. Power harmonics are signals that are synthesized from various frequency components, which are random, appear and disappear very suddenly. When the application of discrete Fourier transform is limited, the advantages of wavelet transform can be fully utilized. After the wave sampling is discrete, the digital signal is processed using wavelet transform, so as to achieve accurate determination of harmonics.

2. Mathematical Theory of Wavelet

2.1. Mathematical Concepts of Wavelet

The physical concept of wavelet is a "small wave", which refers to a function with zero mean and localized energy in the domains of time and frequency. Its waveform is a positive and negative alternating oscillation waveform with attenuation at both ends of zero.

Let $\psi(t) \in L^2(R)$ represent a square integrable real number space [4], that is, a signal space which is limited energy, and whose Fourier transform is $\hat{\psi}(\omega)$. When $\hat{\psi}(\omega)$ can meet the admissible condition

$$C_\psi = \int_{-\infty}^{+\infty} [|\psi(\omega)/\omega|^2] d\omega < +\infty \tag{1}$$

$\psi(t)$ is called a basic wavelet or mother wavelet. Then after scaling and translation of the basic wavelet $\psi(t)$, a wavelet sequence can be obtained.

For a continuous wavelet sequence, the wavelet sequence is

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \qquad a,b \in R \tag{2}$$

Where a is the scale factor and b is the translation factor, a is used to adjust the frequency range covered by the wavelet, b is used to adjust the time domain position of the wavelet, and the coefficient $\frac{1}{\sqrt{a}}$ is used to achieve the normalization of the wavelet energy. The two parameters of a and b are the key to wavelet transform. The shape of the mother wavelet function is fixed, and only through scaling and time translation can it be compared with the signal waveform to extract the characteristics of the signal.

If the signal $f(t) \in L2\ (R)$ and $\psi(t) \in L^2(R)$, and $\psi(t)$ meet the permissive conditions, which is

$$C_\psi = \int_{-\infty}^{+\infty} [|\psi(\omega)/\omega|^2] d\omega < +\infty \tag{3}$$

Then the continuous wavelet transform (CWT) and its inverse transform can be defined as:

$$WT_x(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt \quad f(t) = \frac{1}{c_\psi}\int_0^{\infty}\frac{da}{a^2}\int_{-\infty}^{\infty} WT_f(a,\tau)\frac{1}{\sqrt{a}}\psi\left(\frac{t-\tau}{a}\right)d\tau \tag{4}$$

2.2. Introduction of Multi-Resolution Analysis and Mallat Algorithm

In 1988, S. Mallat put forward the concept of Multi-Resolution when constructing orthogonal basis [5], then explained the multi-resolution analysis characteristics of wavelet from the concept of space. He unify all previous construction methods of orthogonal wavelet bases, and gave the construction method of orthogonal wavelet and fast algorithm of orthogonal wavelet transform, that is, Mallat algorithm.

In the Mallat algorithm, wavelet decomposition is achieved through a series of filtering, as shown in Figure 1. The filters in the box are replaced by corresponding coefficients. First, the signal S passes through the high-pass filter h1 to obtain the detail coefficient cD1; after passing the low-pass filter h0, the smoothing approximation coefficient cA1 is obtained. Because the filtered signal band becomes narrower, according to the sampling law, only half of the sampling points can represent all the information of the original signal, so at this time, "two decimation" is performed on cD1 and cA1, that is, every interval one is removed to halve its length.

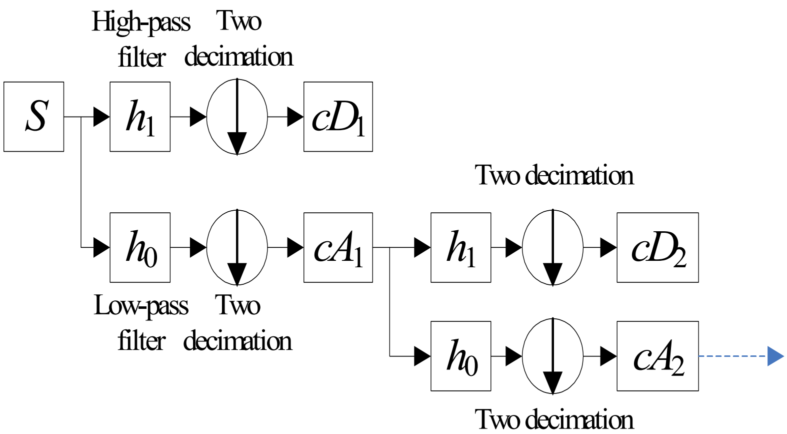

Figure 1 Multi-resolution analysis algorithm

After the above operation, the discrete wavelet coefficient cD1 is obtained when the scale a = 1/2. If the coefficients on other scales are solved, it is necessary to repeat the filtering and two decimation of S for the smoothing approximation coefficient cA1 to obtain the next-level detail coefficient cD2 and the smoothing approximation coefficient cA2. Repeatedly, a binary discrete wavelet transform can be calculated. Since it is a binary discrete wavelet transform here, the first scale corresponds to a = 1/2, the second scale corresponds to a = 1/4, and so on.

Figure 2 shows the Mallat reconstruction algorithm. The reconstruction process of the original signal includes "two interpolation" and filtering. Two-interpolation is the inverse of two decimation, that is, a zero is filled between every two adjacent sampling points of the sequence to double the length of the signal. The filter coefficients are g1 and g0 mentioned above. After the detailed coefficient cDi and smoothing coefficient cAi of the i-th stage are subjected to two interpolation and filtering, the smoothing coefficient of the i+1th stage is obtained by addition. This recursion can finally get the original signal according to cD1 and cA1.

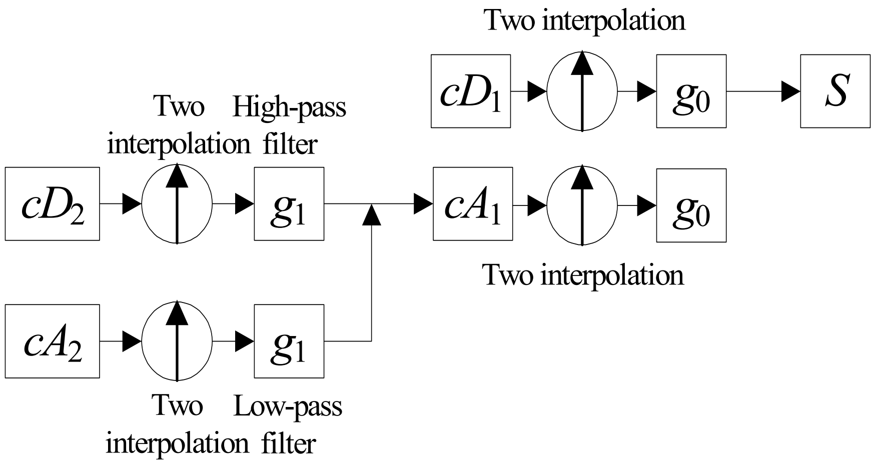

Figure 2 Mallat reconstruction algorithm

2.3. Wavelet Packet Analysis

Multi-resolution analysis has the following characteristics: it can effectively perform the time-frequency decomposition on the signal, but its frequency resolution is poor in the high frequency band, meanwhile its time resolution is poor in the low frequency band, because its scale changes in binary. That is, the frequency band division performed of the signal is an exponential equal interval division (with an equal-Q structure). As shown in Figure 3. The multi-resolution analysis only further decomposes the low-frequency part, and the high-frequency part is not considered. In actual applications, the signal to be analyzed is often a low-frequency signal [6].

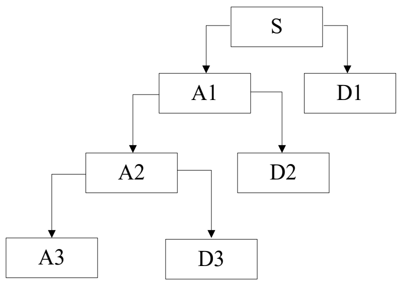

Figure 3 Three-layer multi-resolution analysis

A more detailed analysis method for signals is Wavelet Packet Analysis. Wavelet Packet Analysis can divide the frequency band into multiple levels, and further decomposes the high-frequency parts that which can not subdivided by the multi-resolution analysis. Meanwhile it can analyze the characteristics of the analyzed signal. Wavelet Packet Analysis can adaptively select the corresponding frequency band to match the signal spectrum, thereby improving the time-frequency decomposition rate. Therefore, the wavelet packet can have a wider application value. The decomposition of four-layer wavelet packet tree is shown in the following figure 4.

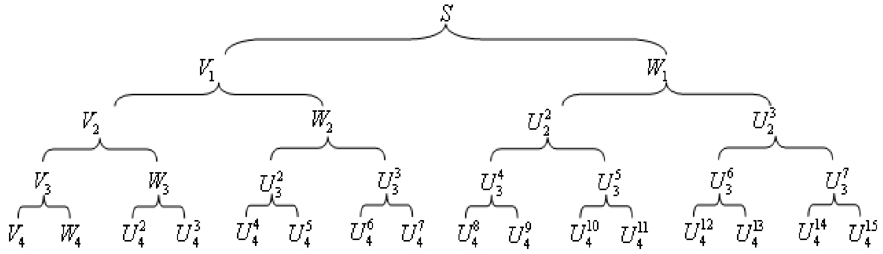

Figure 4 Decomposition of four-layer wavelet packet tree

3. Harmonic Detection Method Based on Wavelet Transform

3.1. Harmonic Detection Method Based on Multi-Resolution Analysis

When using multi-resolution analysis method to detect a abrupt and singular signals of the harmonic signals, first, we need to choose a reasonable number of wavelet decomposition layers and divide the frequency band of the signal correctly. There is a principle of frequency band division: position the fundamental frequency of the signal as the center of the lowest sub-band try to, so as to limit the influence of the fundamental frequency component on other sub-bands.

Let the sampling frequency is f_s, and the fundamental frequency is f_f. In this case, the number of frequency band divisions can be obtained by taking an integer as follows:

$$p = \log_2\left(\frac{f_s}{f_f}\sqrt{\frac{1}{8}}\right) + 0.5 \qquad (5)$$

Since the fundamental signal of China's power system is 50HZ, so the sampling frequency can be set as 6.4KHZ, from the above formula, the number of sub-bands that can be set as 6, which means that the signal should be subjected to 5-layer multi-resolution analysis. Figure 5 shows the schematic diagram of frequency band division. under this circumstances, the fundamental frequency can be located at the center of the lowest sub-band.

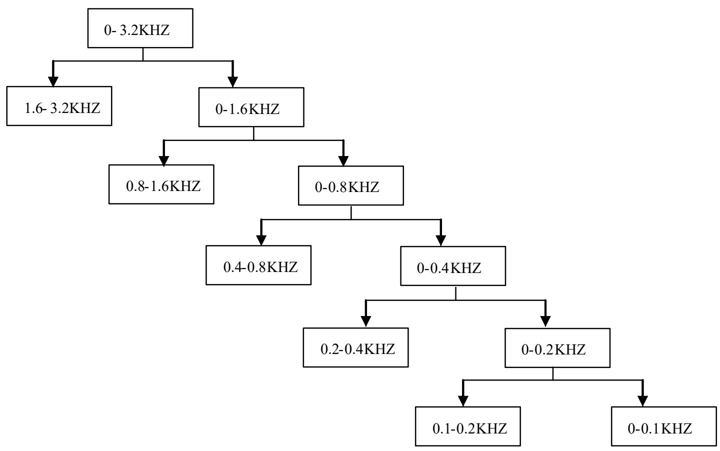

Figure 5 Schematic diagram of frequency band division

3.2. Harmonic Detection Method Based on Wavelet Packet Algorithm

After we decompose the wavelet packet transform, each frequency band will have a same bandwidth, that is, the harmonic order contained in each frequency band is the same. In theory, as long as the number of decompositions is large enough, the harmonic measurements of the frequency range can be greatly improved as much as possible.

For a signal with a sampling frequency of ω, the detectable frequency band is (0, ω/2).In the wavelet packet transform, after passing the first-order image filter, the entire frequency band is divided into the high-frequency band HP (ω/4, ω/2) and the low-frequency band LP (0, ω/4), the frequency of the LP signal (0, ω/4) is unchanged after down-sampling, and after the next level of filter decomposition, this frequency band can be divided into two frequency bands, which are LP (0, ω/8), HP signal (ω/8, ω/4), and down-sampling processing of the HP signal, so that the HP signal (ω/4, ω/2) is converted into a frequency band of (0, ω/4) signal, and then passed through the next-stage image filter, the frequency band can be decomposed into two frequency bands, which are (0, ω/8) and (ω/8, ω/4), respectively corresponding to (3ω/8, ω/2) and (ω/4, 3ω/8)in the original signal, so that the binary division of the high frequency band is realized. Constantly repeating the binary division of the low-frequency and high-frequency bands can achieve uniform subdivision of the frequency band.

However, according to the above analysis, it can be seen that the frequency band division obtained under the existing filter bank structure is not continuously divided in order. The low-frequency filter may correspond to high-frequency signals, while the high-pass filter may correspond to low-frequency signals. For example, after passing the first-stage filter bank, high-frequency signals are down-sampled. According to the above rules, higher frequencies are converted to lower frequencies, and lower frequencies are converted to higher frequencies. The structure of a traditional wavelet filter is shown in Figure 6. we can see shat, the signal of the low-pass filter of the first stage corresponds to the high-frequency signal of the original signal, while the signal of the high-pass filter of the next stage corresponds to the low-frequency signal of the original signal.

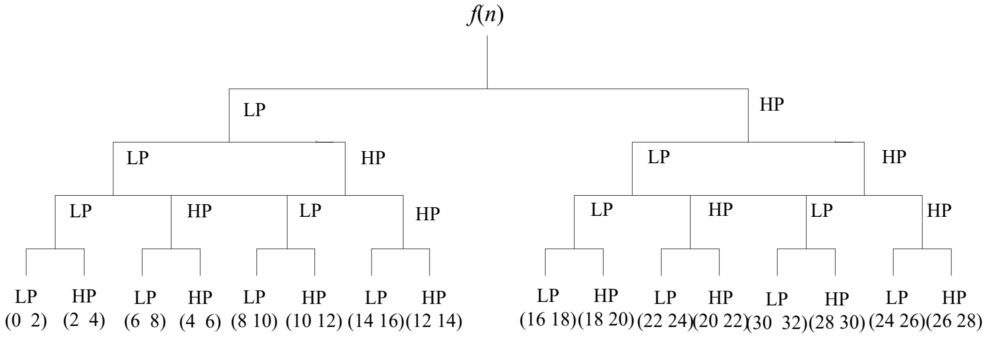

Figure 6 The structure of a traditional wavelet filter

It can be seen that this wavelet filter structure will encounter the situations that the size of each frequency band mark cannot completely correspondand the corresponding frequency band, that is, the signal of the low frequency band may be decomposed into high frequency band, while the signal of the high frequency band may decompose low frequency bands. This will bring difficulties to the frequency analysis and feature extraction of harmonics, and it will not be able to intuitively judge each harmonic, which will affect the use of wavelet packets.

To solve the problems, this paper proposes a new wavelet filter structure, as shown in Figure 7.

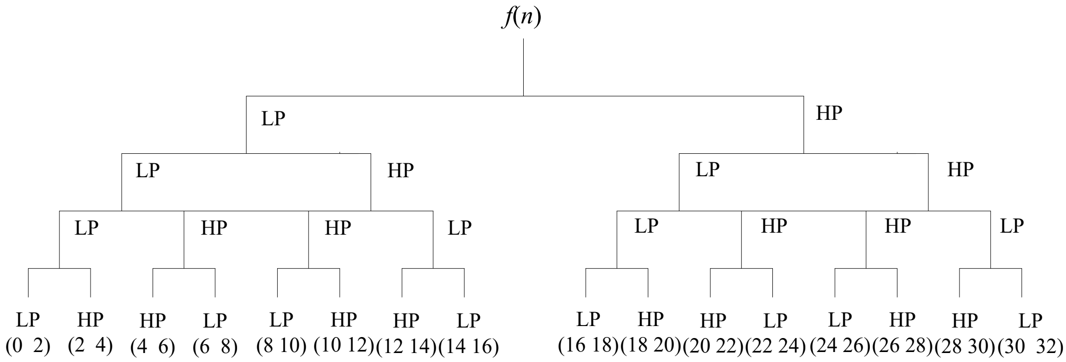

Figure 7 The new wavelet filter structure

In this filter structure shown in figure 7, there is still a case where the size of each frequency band mark and the corresponding frequency band frequency cannot completely correspond. That is, the signal of the low frequency band may be decomposed into high frequency band, while the signal of the high frequency band may be decomposed into low frequency band, and there is no clear corresponding rule for the arrangement of the upper and lower decomposition layer filters. Therefore, a new filter bank structure is proposed in this paper, as shown in Figure 8.

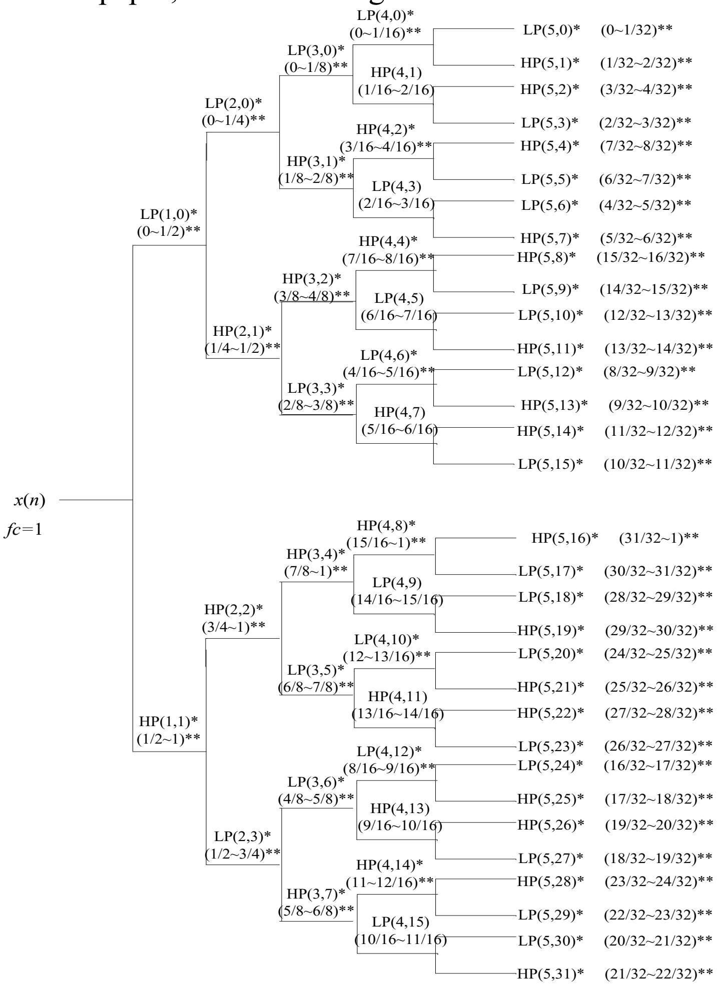

Figure 8 The new filter bank structure and frequency band division

The filter combination structure has a clear arrangement rule. After the signal is decomposed into high and low frequency bands by a pair of high and low frequency filters on the first layer, when performing k-layer decomposition, if the k-1 layer is a low-pass filter , Then the order of the filters in the k-th layer is low first and then high. Similarly, if the k-1 layer is a high-pass filter, the order of filters in the k-th layer is high first and then low. It is consistent with the arrangement order of the nodes in the wavelet packet decomposition, and at the same time, it can easily determine the corresponding signal frequency band.

4. Conclusion

In this paper, we introduces the main application method of wavelet transform in power system harmonic detection, and proposes a new design method of wavelet filter. The design of wavelet filter in this paper can orderly arrange the signal frequency bands and make wavelet transform better applied.

Acknowledgements

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